Equivalent Representations of Lossy Nonuniform Transmission Lines

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***Abstract* —Equivalent transformations, which were recently derived for mixed lumped and distributed circuits, may be extended to circuits consisting of lumped reactances, resistors, and lossy transmission lines. It is shown that circuits consisting of a cascade connection of lumped element sections and lossy uniform transmission lines are equivalent to circuits consisting of a cascade connection of lossy nonuniform transmission lines, lumped elements, and ideal transformers. Furthermore, by considering the limiting case of these transformations, equivalent transformations for circuits consisting of a cascade connection of lumped reactances, resistors, and nonuniform transmission lines are obtained. Exact equivalent circuits of lossy even-order binomial form transmission lines are derived from these equivalent transformations.**

I. Introduction

Recently, new equivalent transformations for mixed lumped and distributed circuits have been obtained based on Kuroda's identities. By using these new transformations, a class of nonuniform transmission lines may be derived with the circuits consisting of cascade connections of lumped reactive elements, uniform transmission lines, negative lumped reactive elements, and ideal transformers. The network functions of these nonuniform transmission lines can be obtained exactly without solving the telegrapher's equation [1], [2].

In microwave technology, lossy nonuniform transmission lines such as RC tapered transmission lines are useful in component design, and the analysis of mixed lumped and lossy distributed circuits may be necessary for the design of matching sections, filters, and so on [3], [4].

In this paper, we discuss equivalent transformations for circuits consisting of mixed lumped and lossy distributed circuits. First, we give the formal equivalent transformation for a circuit consisting of a cascade connection of a parallel (series) element and a lossy uniform transmission line (LUE) of line length l/n. This formal equivalent transformation may be applied n-times to a circuit consisting of a cascade connection of a parallel (series) element and a lossy transmission line of line length l. By considering the limit case of $n \to \infty$ and giving a certain condition between the parallel (series) element values and the primary constants of the LUE, we show that the equivalent circuit of a cascade connection of a lumped reactance, resistor, and a LUE is given as a circuit consisting of a cascade connection of a lossy nonuniform transmission line, lumped reactance, resistor, and an ideal transformer.

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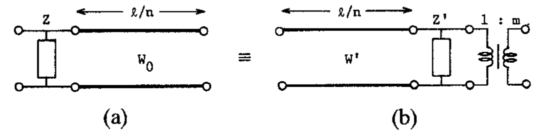


Fig. 1. The formal equivalent transformation for the circuit of a shunt section and a LUE.

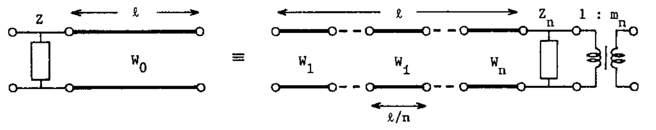
Next, general transformations for the mixed lumped and lossy nonuniform distributed circuits are given. We show that the equivalent circuit of a cascade connection of lumped reactive and resistive elements and a lossy nonuniform transmission line becomes one consisting of a cascade connection of a lossy nonuniform transmission line, lumped reactive and resistive elements, and an ideal transformer. If a characteristic impedance distribution $W(x)$ of an original lossy nonuniform transmission line can be integrated, a characteristic impedance distribution $z(x)$ of a transformed nonuniform transmission line may be uniquely obtained using $W(x)$. By using these integral formulations again and again, we may obtain the equivalent circuits of even-order lossy binomial form nonuniform transmission lines. The equivalent circuits of RC transmission lines and GL transmission lines are obtained as the special cases of these equivalent transformations.

II. Equivalent Transformations for Mixed Lumped and Lossy Distributed Circuits

A. Transformations for Circuits Consisting of a Cascade Connection of a Parallel Lumped RL in Series and a Lossy Unit Element

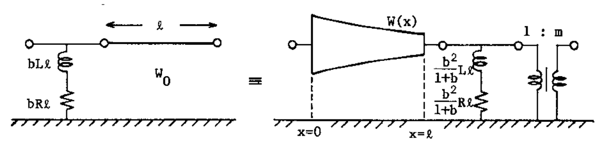
The equivalent representation of the circuit consisting of a cascade connection of a shunt section and a lossy unit element (LUE) shown in Fig. 1(a) is given as a cascade connection of a LUE, a shunt section, and an ideal transformer, as shown in Fig. 1(b). In Fig. 1, Z and Z' are the impedances of shunt sections, W_0 and W' are the characteristic impedances of LUE's, m is the transformation ratio of the ideal transformer, and l/n is the line length of a LUE.

The element values of the transformed circuit are given



(a) (b)

Fig. 2. The formal equivalent transformation for the circuit of a shunt section and a LUE of line length of l.



(a) (b)

Fig. 3. The shunt section–LUE transformation.

as follows:

$$W' = W_0 \frac{Z}{Z + pW_0} \tag{1}$$

$$Z' = \frac{Z^2}{Z + pW_0} \tag{2}$$

$$m = \frac{Z + pW_0}{Z} \tag{3}$$

where

$$W_0 = \sqrt{\frac{R + sL}{G + sC}} \tag{4}$$

and

$$p = \tanh \gamma \left(\frac{l}{n} \right). \tag{5}$$

Where R, L, C, and G are primary constants of the LUE, γ is the propagation constant given as

$$\gamma = \sqrt{(R + sL)(G + sC)} \tag{6}$$

and s denotes the complex frequency.

If a lossless UE is considered and a parallel element is a single short-circuited stub, we obtain Kuroda's Identity [5].

The equivalent transformation shown in Fig. 1 can be applied n-times (n: integer) to a circuit consisting of a cascade connection of a shunt section and a LUE whose line length is l, as shown in Fig. 2. The transformed circuit consists of a cascade connection of LUE's with each line length equal to l/n, a parallel element section, and an ideal transformer. The element values of the transformed circuit are given as follows:

$$W_i = \frac{W_0 \left(\dfrac{Z}{npW_0} \right)^2}{\left(\dfrac{Z}{npW_0} + \dfrac{i-1}{n} \right)\left(\dfrac{Z}{npW_0} + \dfrac{i}{n} \right)} \qquad (i = 1, 2, \cdots, n) \tag{7}$$

$$Z_n = \frac{Z \dfrac{Z}{npW_0}}{1 + \dfrac{Z}{npW_0}} \tag{8}$$

$$m_n = 1 + \frac{1}{\dfrac{Z}{npW_0}}. \tag{9}$$

The characteristic impedance of W_i in (7) and the transformer ratio m_n in (9) are functions of Z and p, so it is difficult to realize the circuit shown in Fig. 2(b), physically. But by setting the value of Z to be an appropriate one and considering the limit case (n to infinity), we can obtain physically realizable transformed circuits.

One of the suitable choices of parallel element is

$$Z = b(R + sL)l \qquad (b\text{: constant}) \tag{10}$$

where R and L are primary constants of the original LUE.

Here, we define the coordinates x of the ith LUE of the transformed circuit as follows [1]:

$$x = \frac{i}{n} l. \tag{11}$$

By substituting (10) and (11) into (7)–(9), allowing n to approach infinity, and using (6) we obtain

$$\lim_{n \to \infty} \frac{Z}{npW_0} = \lim_{n \to \infty} \frac{Z}{n \tanh \gamma (l/n) W_0} = \frac{Z}{\gamma l W_0} = \frac{Z}{(R + sL)l} = b \tag{12}$$

$$\lim_{n \to \infty} W_i = \frac{W_0}{\left(1 + \dfrac{1}{b} \dfrac{x}{l}\right)^2} \equiv W(x) \tag{13}$$

$$\lim_{n \to \infty} m_n = 1 + \frac{1}{b} \equiv m \tag{14}$$

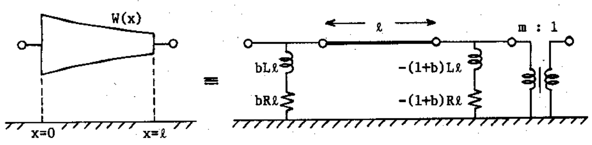
and

$$\lim_{n \to \infty} Z_n = \frac{Z}{m} = \frac{b^2}{1 + b}(R + sL)l. \tag{15}$$

At the limit, the lossy cascaded transmission lines (CTL's) become a nonuniform transmission line whose characteristic impedance distribution is $W(x)$, the impedance of the transformed parallel element becomes Z/m, and the transformation ratio of the ideal transformer becomes constant. This equivalent circuit is physically realizable.

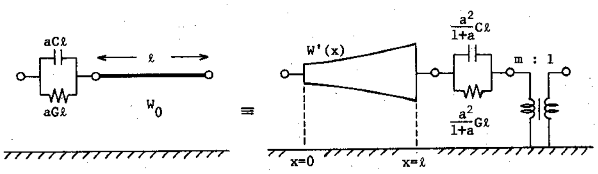
Thus, the equivalent circuit of the cascade shown in Fig. 3(a) is a circuit consisting of a cascade connection of a lossy nonuniform transmission line whose characteristic impedance distribution is $W(x)$, a series RL element in parallel, and an ideal transformer, as shown in Fig. 3(b).

By using this equivalent transformation (shunt section–LUE transformation), it may be shown that the equivalent circuit of the nonuniform transmission line whose characteristic impedance is $W(x)$ in (13) may be expressed as a circuit shown in Fig. 4.



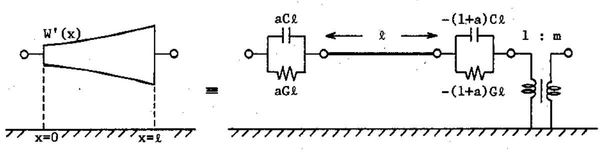
(a) (b)

Fig. 4. The equivalent circuit of a lossy nonuniform transmission line of $W(x)$ given in (13).



(a) (b)

Fig. 5. The series section–LUE transformation.



(a) (b)

Fig. 6. The equivalent circuit of the lossy nonuniform transmission line of $W'(x)$ given in (16).

B. Transformations for Circuits Consisting of a Cascade Connection of a Series Lumped GC in Parallel and a Lossy Unit Element

In the same manner as described in Section II-A, we obtain the dual transformation to the circuit consisting of a cascade connection of a series section of a LUE as shown in Fig. 5, where a is a constant, G and C are primary constants of LUE, and $W'(x)$ is given by

$$W'(x)=W_0\left(1+\frac{1}{a}\frac{x}{l}\right)^2. \tag{16}$$

By using this equivalent transformation, the equivalent circuit of the nonuniform transmission line with characteristic impedance distribution $W'(x)$ in (16) is expressed by the one shown in Fig. 6.

III. EQUIVALENT TRANSFORMATIONS FOR MIXED LUMPED AND LOSSY NONUNIFORM DISTRIBUTED CIRCUIT

The shunt section–LUE transformation shown in Fig. 1 may be applied n-times to the circuit consisting of a cascade connection of a parallel impedance Z and CTL's, where the characteristic impedance of the ith LUE of the CTL's is given as

$$W_i=h_i\sqrt{\frac{R+sL}{G+sC}} \qquad (h_i\text{: real constants}, \quad i=1,2,\cdots,n) \tag{17}$$

and the line length of each LUE is l/n. This equivalent transformation (shunt section–CTL's transformation) is shown in Table I.

TABLE I
THE SHUNT SECTION–CTL'S TRANSFORMATION

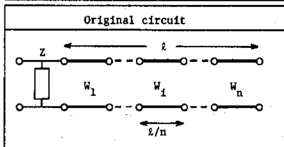
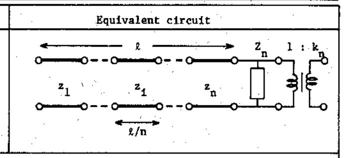
Original circuit	Equivalent circuit
	
Formulas	
$k_j=1+\frac{p}{Z}\sum_{i=1}^{j}W_i$ $(j=1,2,\text{---},n)$, $k_0=1$ $z_j=\frac{W_j}{k_{j-1}k_j}$ $(j=1,2,\text{---},n)$, $Z_n=\frac{Z}{k_n}$ W_i, z_i : characteristic impedance Z, Z_n : impedance of parallel section $p=\tanh\gamma(\ell/n)$: Richards' variable	

TABLE II
THE SHUNT SECTION–NONUNIFORM TRANSMISSION LINE TRANSFORMATION

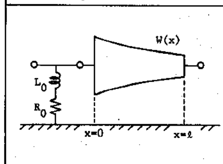
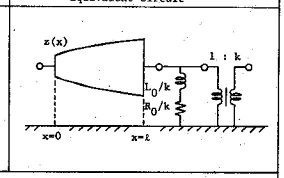
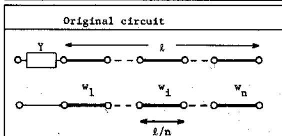
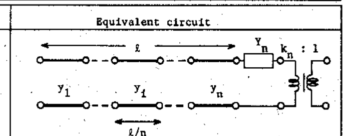
Original circuit	Equivalent circuit
	
Formulas	
$k(x)=1+\frac{1}{bW_0}\int_0^{x/\ell}W(\lambda)d(\frac{\lambda}{\ell})$, $z(x)=\frac{W(x)}{k(x)^2}$, $k=k(x)\Big\|_{x=\ell}$ $W(x)=W_0\{1+\alpha_1(\frac{x}{\ell})+\alpha_2(\frac{x}{\ell})^2+\text{---}\}$, $W_0=\sqrt{\frac{R+sL}{G+sC}}$ $L_0=bL\ell$ $R_0=bR\ell$	

TABLE III
THE SERIES SECTION–CTL'S TRANSFORMATION

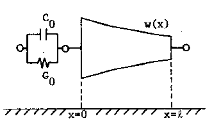
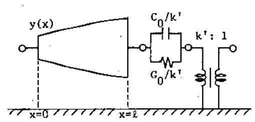
Original circuit	Equivalent circuit
	
Formulas	
$k_j=1+\frac{p}{Y}\sum_{i=1}^{j}w_i$ $(j=1,2,\text{---},n)$, $k_0=1$ $y_j=\frac{w_j}{k_{j-1}k_j}$ $(j=1,2,\text{---},n)$, $Y_n=\frac{Y}{k_n}$ w_i, y_i : characteristic admittance Y, Y_n : admittance of series section $p=\tanh\gamma(\ell/n)$: Richards' variable	

Here, we assume that the shunt section is constructed as a series RL whose element values are proportional to the primary constants of each LUE of the original CTL's. Proceeding to the limit $n\to\infty$, we obtain the equivalent transformations of cascade connections of shunt sections and nonuniform transmission lines shown in Table II.

In Table II, $W(x)$ and $z(x)$ are the characteristic impedance distributions of the lossy nonuniform transmission lines. Formulas in Table II can be obtained by the same technique described in a previous paper [2] so that the derivation may be omitted here.

The dual transformation for a series admittance Y and CTL's is shown in Table III. We assume that the series

TABLE IV
THE SERIES SECTION–NONUNIFORM TRANSMISSION LINE TRANSFORMATION

Original circuit	Equivalent circuit
	
Formulas	
$k'(x) = 1 + \frac{W_0}{a}\int_0^{x/\ell} w(\lambda)d(\frac{\lambda}{\ell})$, $y(x) = \frac{w(x)}{k'(x)^2}$, $k' = k'(x)\Big\|_{x=\ell}$ $w(x) = W_0^{-1}\{1 + \alpha_1'(\frac{x}{\ell}) + \alpha_2'(\frac{x}{\ell})^2 + ---\}$, $W_0 = \sqrt{\frac{R+sL}{G+sC}}$ $C_0 = aC\ell$ $G_0 = aG\ell$	

section is constructed with a parallel combination of a lumped capacitor and a resistor whose element values are proportional to the primary constants of each LUE of the original CTL's. Proceeding to the limit $n \to \infty$, we obtain the equivalent transformation of the circuit consisting of a cascade connection of a series section and a nonuniform transmission line as shown in Table IV. In Table IV, $w(x)$ and $y(x)$ are the characteristic admittance distributions.

IV. EQUIVALENT CIRCUITS OF LOSSY BINOMIAL FORM NONUNIFORM TRANSMISSION LINES

By using the transformations shown in Tables II and IV, we may obtain the equivalent circuits of lossy nonuniform transmission lines. As an example, we show the equivalent circuits of lossy binomial form nonuniform transmission lines.

A. Second-Order Lossy Binomial Form Transmission Line

The equivalent circuit of a second-order binomial form nonuniform transmission line is given in Fig. 4. Here, for simplicity of notation, we replace the impedance Z in (10) with Z_1, where

$$Z_1 = b_1(R+sL)l. \tag{18}$$

In this case,the formulas in Table II are expressed as

$$k_1(x) = 1 + \frac{1}{b_1 W_0}\int_0^{x/l} W_0 d\left(\frac{\lambda}{l}\right) = 1 + \frac{1}{b_1}\frac{x}{l} \tag{19}$$

$$k_1 = k_1(x)|_{x=l} = 1 + \frac{1}{b_1} \tag{20}$$

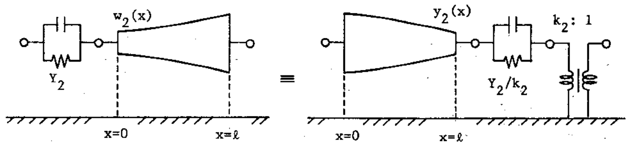
and

$$z_1(x) = \frac{W_0}{k_1(x)^2} = \frac{W_0}{\left(1 + \frac{1}{b_1}\frac{x}{l}\right)^2}. \tag{21}$$

B. Fourth-Order Lossy Binomial Form Transmission Line

We consider the transformation shown in Table IV to the circuit shown in Fig. 7 under the following conditions:

$$Y_2 = a_2(G+sC)l \tag{22}$$



(a) (b)

Fig. 7. The equivalent transformation of series section–nonuniform transmission line of $w_2(x)$ given in (23).

and

$$w_2(x) = W_0^{-1}\left(1 + \frac{1}{b_1}\frac{x}{l}\right)^2. \tag{23}$$

If the coefficients a_2 and b_1 satisfy the relation

$$b_1 = 3a_2 \tag{24}$$

we obtain

$$k_2(x) = 1 + \frac{W_0}{a_2}\int_0^{x/l} w_2(\lambda)d\left(\frac{\lambda}{l}\right) = \left(1 + \frac{1}{b_1}\frac{x}{l}\right)^3 \tag{25}$$

$$k_2 = \left(1 + \frac{1}{b_1}\right)^3 = k_1^3 \tag{26}$$

and

$$y_2(x) = \left[W_0\left(1 + \frac{1}{b_1}\frac{x}{l}\right)^4\right]^{-1}. \tag{27}$$

The characteristic impedance distribution $1/y_2(x)$ is the fourth-order binomial form.

C. Sixth-Order Lossy Binomial Form Transmission Line

We set the characteristic impedance distribution as

$$W_3(x) = \frac{1}{y_2(x)} = W_0\left(1 + \frac{1}{b_1}\frac{x}{l}\right)^4 \tag{28}$$

and again apply the equivalent transformation shown in Table II by setting

$$Z_3 = b_3(R+sL)l. \tag{29}$$

Under the condition

$$b_1 = 5b_3 \tag{30}$$

we obtain the following relations:

$$k_3(x) = \left(1 + \frac{1}{b_1}\frac{x}{l}\right)^5 \tag{31}$$

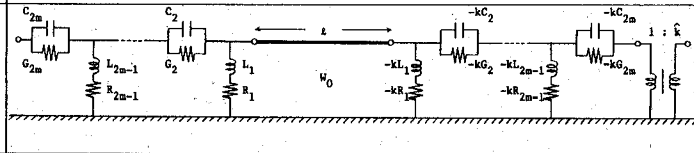
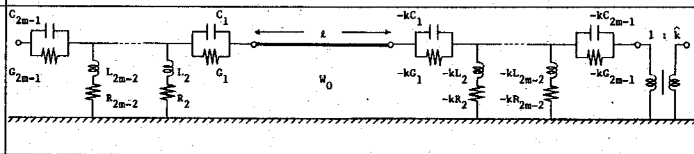
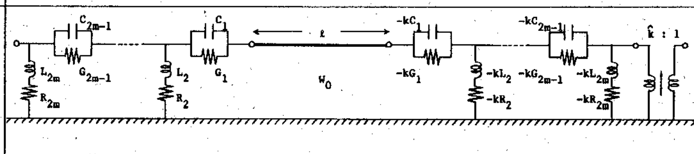
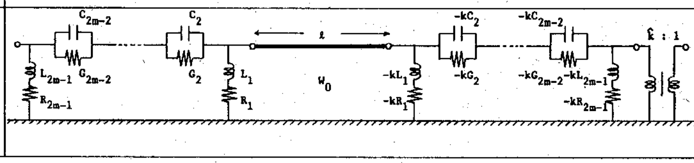
$$k_3 = \left(1 + \frac{1}{b_1}\right)^5 = k_1^5 \tag{32}$$

$$z_3(x) = \frac{W_0}{\left(1 + \frac{1}{b_1}\frac{x}{l}\right)^6}. \tag{33}$$

Here, we may obtain the equivalent circuit of sixth-order lossy binomial form transmission line.

We may carry out these procedures in a sequential manner and obtain the equivalent representations of even-

TABLE V
EQUIVALENT CIRCUITS OF EVEN-ORDER LOSSY BINOMIAL FORM NONUNIFORM TRANSMISSION LINES

Characteristic impedance distribution	Equivalent circuit	Formulas
$W(x) = W_0\left(1 + \frac{1}{b}\frac{x}{\ell}\right)^{4m}$		$L_{2i-1} = bL\ell/(4i-3)$ $R_{2i-1} = bR\ell/(4i-3)$ $(i=1,2,---,m)$ $C_{2i} = bC\ell/(4i-1)$ $G_{2i} = bG\ell/(4i-1)$ $(i=1,2,---,m)$ $k = 1 + 1/b$, $\hat{k} = k^{2m}$
$W(x) = W_0\left(1 + \frac{1}{a}\frac{x}{\ell}\right)^{4m-2}$		$L_{2i} = aL\ell/(4i-1)$ $R_{2i} = aR\ell/(4i-1)$ $(i=1,2,---,m-1)$ $C_{2i-1} = aC\ell/(4i-3)$ $G_{2i-1} = aG\ell/(4i-3)$ $(i=1,2,---,m)$ $k = 1 + 1/a$, $\hat{k} = k^{2m-1}$
$W(x) = \frac{W_0}{\left(1 + \frac{1}{a}\frac{x}{\ell}\right)^{4m}}$		$L_{2i} = aL\ell/(4i-1)$ $R_{2i} = aR\ell/(4i-1)$ $(i=1,2,---,m)$ $C_{2i-1} = aC\ell/(4i-3)$ $G_{2i-1} = aG\ell/(4i-3)$ $(i=1,2,---,m)$ $k = 1 + 1/a$, $\hat{k} = k^{2m}$
$W(x) = \frac{W_0}{\left(1 + \frac{1}{b}\frac{x}{\ell}\right)^{4m-2}}$		$L_{2i-1} = bL\ell/(4i-3)$ $R_{2i-1} = bR\ell/(4i-3)$ $(i=1,2,---,m)$ $C_{2i} = bC\ell/(4i-1)$ $G_{2i} = bG\ell/(4i-1)$ $(i=1,2,---,m-1)$ $k = 1 + 1/b$, $\hat{k} = k^{2m-1}$

order lossy binomial form transmission lines. The four types of equivalent circuits of even-order lossy binomial form transmission lines are shown in Table V. These equivalent circuits consist of cascaded ladder networks constructed with parallel lumped GC and series lumped RL arms, a lossy uniform transmission line, a ladder network with negative lumped element values, and an ideal transformer.

For the special case of $R = G = 0$ in the primary constants, the element values of the equivalent circuits become lossless ones [1], [6]. For another practical case of $L = G = 0$ in the primary constants, the original transmission lines become even-order RC binomial form transmission lines, and the equivalent circuit of this transmission line consists of a cascade connection of a lumped RC ladder network, an RC uniform transmission line, a lumped negative RC ladder network, and an ideal transformer. Similarly, if we set $C = R = 0$ in the primary constants, we obtain the equivalent circuits of even order GL binomial form transmission lines.

V. CONCLUSIONS

We have shown equivalent transformations for the circuits consisting of mixed lumped and lossy nonuniform transmission lines.

First, we showed the equivalent transformations for the circuits consisting of a cascade connection of a parallel element section of lumped RL series impedance and a LUE, and for the dual case. Then by repeating these procedures for the cascade connection of lumped reactances and resistors and CTL's, we showed the equivalent transformations for the mixed lumped and lossy nonuniform transmission lines in the limit case.

As an example, we showed the equivalent circuits of even-order lossy binomial form nonuniform transmission lines. By using these equivalent circuits, the exact network functions of lossy nonuniform transmission lines can be derived without solving the telegrapher's equation.

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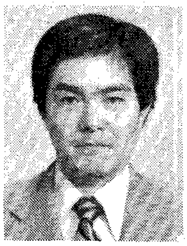
Isao Endo was born in Fukushima, Japan, on August 17, 1949. He received the B.E. and M.E. degrees in electronics engineering from Ibaraki University, Hitachi, Japan, in 1973 and 1975, respectively.



From 1975 to 1980 he was a Research Associate and from 1980 to 1983 he was a Lecturer in the Department of Electric Engineering, Ibaraki Technical College, Katsuta, Japan. He is now an Associate Professor. His research interests include circuit theory and matching network design.

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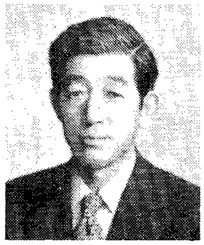
Yoshiaki Nemoto (S'72–M'73) was born in Sendai City, Miyagiken, Japan, on December 2, 1945. He received the B.E., M.E., and Ph.D. degrees from Tohoku University, Sendai, Japan, in 1968, 1970, and 1973, respectively.

Since 1973 he has been a Research Associate with the Faculty of Engineering, Tohoku University. He has been engaged in research works in distributed networks and computer networks using satellites. He is co-recipient of the 1982 Microwave Prize from the IEEE Microwave Theory and Techniques Society.

Dr. Nemoto is a member of the Institute of Electronics and Communication Engineers of Japan.



Risaburo Sato (SM'62–F'77) was born in Furukawa City, Miyagiken, Japan, on September 23, 1921. He received the B.E. and the Ph.D. degrees from Tohoku University, Sendai, Japan, in 1944 and 1952, respectively.

From 1949 to 1961 he was an Assistant Professor at Tohoku University, and in 1961 he became a Professor in the Department of Electrical Communications at the same university. Since 1973 he has been a Professor in the Department of Information Science at Tohoku University. From 1969 to 1970 he was an International Research Fellow at Stanford Research Institute, Menlo Park, CA. His research activities include studies of multiconductor transmission systems, distributed transmission circuits, antennas, communication systems, active transmission lines, magnetic and ferroelectric recording, neural information processing, computer networks, and electromagnetic compatibility. He has published a number of technical papers and some books in these fields, including *Transmission Circuit*. He received the Paper Award from the Institute of Electrical Engineers of Japan (IEE of Japan) in 1955, the Kahoku Press Cultural Award in 1963, an award from the Invention Association of Japan in 1966, the Paper Award from the Institute of Electronics and Communication Engineers of Japan (IECE of Japan) in 1980, a Certificate of Appreciation of Electromagnetic Compatibility from the IEEE in 1981, and the Microwave Prize of the Microwave Theory and Techniques Society of IEEE in 1982.

Dr. Sato was the Vice President of IECE of Japan from 1974 to 1976. He has been a member of the Science Council of Japan from 1978 and a member of the Telecommunication Technology Consultative Committee at NTT from 1976. He is a chairman of EMC-S Tokyo Chapter of IEEE and a member of B.O.D. of EMC-S of IEEE. He is also a member of IECE of Japan, IEE of Japan, the Institute of Television Engineers of Japan, and the Information Processing Society of Japan.

High-Frequency Doubler Operation of GaAs Field-Effect Transistors

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***Abstract*—A comprehensive study of single-gate GaAs FET frequency doublers is presented. Special emphasis is placed on exploring high-frequency limitations, while yielding explanations for previously observed lower frequency phenomena as well. Extensive large-signal simulations demonstrate the underlying relationships between circuit performance characteristics and principal design parameters. Verifying experiments include a straight frequency doubler and a self-oscillating doubler, both with output signal frequencies in *Ka*-band. The self-oscillating doubler appears especially attractive, yielding an overall dc-to-RF efficiency of 10 percent. The type of transistor employed in the numerical and experimental examples possesses a gate length of 0.5 μm and a gate width of 250 μm.**

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I. Introduction

EFFORTS currently directed toward increased utilization of the millimeter-wave frequency range are providing a steady incentive to explore potential alternatives to existing means of generating RF power at these frequencies. In the solid-state domain, recent amplifier results indicate that GaAs FET's with subhalf-micron gate lengths are capable of attractive fundamental frequency oscillation up through at least 40 GHz. An appreciable extension in the useful frequency range for RF power generation should, in principle, be readily obtainable by exploiting device nonlinear properties that permit efficient frequency multi-